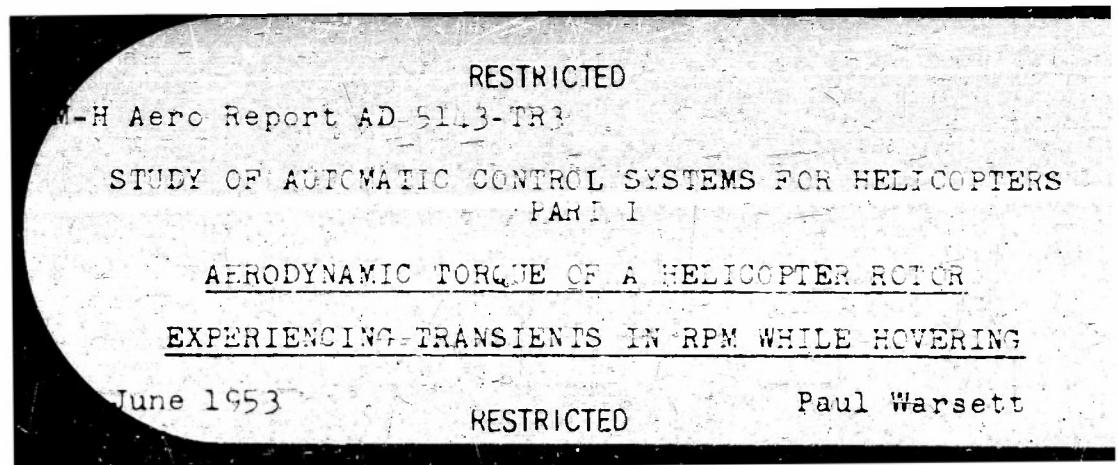
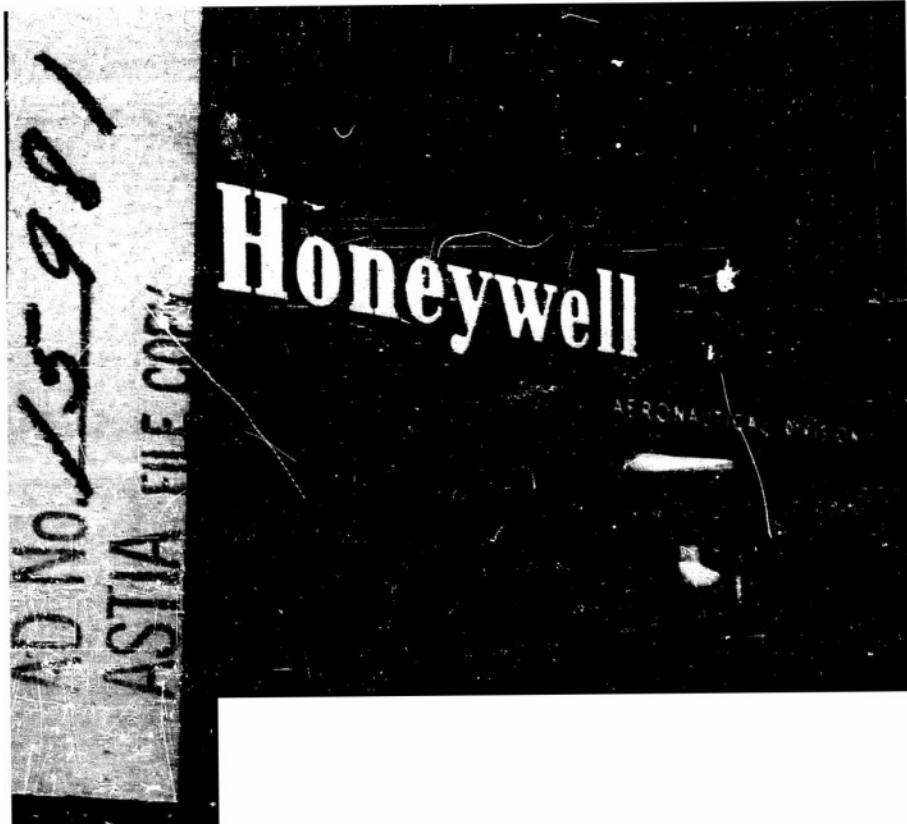


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*Aeronautical Controls*

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**ABSTRACT**

This report contains the development of the theoretical equation for the aerodynamic torque absorbed by a helicopter rotor which is experiencing transient disturbances in RPM while the helicopter is hovering. Inasmuch as the dynamical equation for torque was found to include the effects of the simultaneous transients in blade coning angle and vertical velocity of the aircraft, it was necessary to derive auxiliary equations defining these additional variables, and these equations are also shown here.

The system of equations derived in this report is required in the analytical study of controls for automatic regulation of helicopter rotor RPM.

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## I INTRODUCTION

The analytical study of helicopter rotor RPM variation and the automatic control thereof is based to a large extent on the relationship from dynamics,

$$Q_e - Q_A = I_{eff} \ddot{\Omega} \quad (1)$$

This simply states that the unbalanced torque acting on the rotor gives rise to an angular acceleration (about the rotor shaft) whose magnitude also depends on the effective moment of inertia of the rotating system. The value of the aerodynamic torque  $Q_A$  (sometimes referred to as the 'required' torque or the torque 'absorbed' by the rotor) has been determined theoretically for steady-state rotor operation by many investigators (e.g., Reference (1), equations 2.29 and 4.115). For the case of variable RPM, however, as indicated by equation (1), the expression for  $Q_A$  has apparently not been previously derived. The derivation of this expression was the principal objective of the study reported herein.

The conditions stipulated for the present treatment for the determination of  $Q_A$ , as well as other requirements and assumptions involved in this work, are described in the following:

1. A single, main-rotor type helicopter was selected for analysis.
2. It is presumed that the helicopter was hovering at a point in space, and that it subsequently experienced a vertical gust, a change in collective pitch (say due to pilot action), and/or a change in rotor RPM due to an engine disturbance.
3. It is assumed\* that as a consequence of the above disturbances, the helicopter moves only in a vertical direction, with the tip path plane remaining horizontal.
4. While the induced velocity at the rotor was held to be invariant with time throughout the transient experienced by the helicopter, it was presumed

\* This will be shown in a later work to be a fact rather than an assumption, at least when the tail rotor is disregarded.

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to vary with blade radius in the manner prescribed by the combined blade element-momentum theory. This is in accordance with the conclusions presented in Reference (2). Hence,

$$V = \frac{\rho a \Omega_0 R}{16} \left( -1 + \sqrt{1 + \frac{32 \Omega_0 R}{\rho a R}} \right) \quad (2)$$

Tip loss effects were evaluated by means of the well-known factor  $B$  in computing blade lift forces.

5. Aerodynamic forces were assumed to develop on the rotor blade elements without time lag. These forces were also presumed to be independent of Mach number and Reynolds number variations during the transients.
6. To avoid encountering non-linear differential equations, all variations of parameters were taken to be of very small magnitude.
7. Elastic properties of the rotor were ignored.
8. The drag coefficient at the blade element was taken to be

$$C_D = \delta_0 + \delta_1 \alpha_r + \delta_2 \alpha_r^2 \quad (3)$$

as suggested by Sissings, Bailey, Gessow (see, for example, Reference (3), page 81), and others.

9. The rotor blades were presumed rectangular and untwisted, and the flapping hinge offset  $e$  was considered negligibly small.  $\delta_3$  hinge and drag hinge effects were not considered.

II DERIVATION OF THE TORQUE EQUATION

Following the usual approach for the determination of the aerodynamic forces on the rotor, based on blade element concepts, the diagram of the velocities and elemental forces is considered\*:

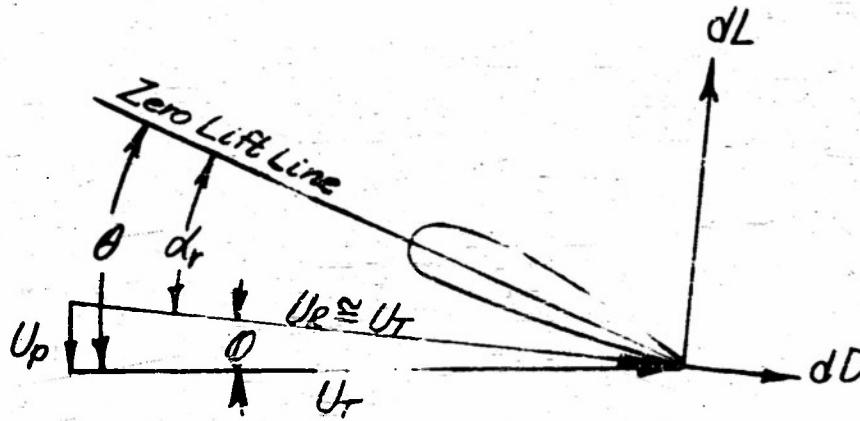


Figure 1

By examination of Figure 1, it is evident that the elemental aerodynamic torque about the rotor axis for the blade element at radius  $r$  is given by

$$dQ_A = r dL \sin \theta + r dD \cos \theta \quad (4)$$

or, since  $U_p$  is everywhere much smaller than  $U_T$ ,

$$dQ_A = r \frac{U_p}{U_T} dL + r dD \quad (5)$$

For the hovering regime,

$$U_T = r \dot{\theta} = r (\dot{\theta}_0 + \dot{\theta}_\Delta) \quad (6)$$

$$U_p = v + r \dot{\beta} + w - \dot{z}_c \quad (7)$$

\* See Section VI for explanation of the symbols employed here.

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Note that equation (7) provides for the possibility of a vertical gust of velocity  $w$  being imposed on the rotor, the (+) sign implies that the gust is directed downwardly. The (-) sign in front of  $\dot{z}_c$  (which is the vertical transient velocity of the helicopter) accounts for the convention that downward movements of the aircraft are positive. The disturbance variables in equation (6) and (7),  $\Omega_\Delta$ ,  $\beta$  and  $\dot{z}_c$ , do not vary with blade radius, and are small quantities. The gust velocity  $w$  which produces the transient effects is also presumed to be small. Also from Figure (1),

$$\alpha_r = \theta - \phi \quad (8)$$

$$= (\theta_0 + \theta_\Delta) - \frac{U_p}{U_f} \quad (9)$$

It is presumed that the lift coefficient is given by the usual expression

$$C_L = \alpha(\alpha_r) \quad (10)$$

Consequently,

$$U Q_A = r \frac{U_p}{U_f} \left[ \frac{\rho c}{2} U_R^2 C_L dr \right] + r \left[ \frac{\rho c}{2} U_e^2 C_0 dr \right] \quad (11)$$

$$= \frac{\rho c}{2} \left[ U_p U_f \alpha(\theta - \phi) + U_f^2 (\delta_0 + \delta_1 \alpha_r + \delta_2 \alpha_r^2) \right] r dr \quad (12)$$

Making the appropriate substitutions, neglecting non-linear and other second order terms, and integrating (to the tip radius  $R$  in the case of drag terms, and to  $BR$  in the case of lift terms),

$$Q_A = \left\{ \alpha \theta_0 \left[ \Omega_0 (A_1 + B_1 \dot{\beta} + C_1 w - C_1 \dot{z}_c) + \Omega_\Delta A_1 \right] \right. \\ \left. - \alpha \left[ E_1 + 2A_1 \dot{\beta} + 2D_1 w - 2D_1 \dot{z}_c \right] \right\}$$

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$$+ B_2 \Omega_0^2 [ \delta_0 + \delta_1 (\theta_0 + \theta_\Delta) + \delta_2 (\theta_0^2 + 2\theta_0 \theta_\Delta) ]$$

$$+ B_2 (2\Omega_0 \dot{\Omega}_0) [ \delta_0 + \delta_1 \theta_0 + \delta_2 \theta_0^2 ]$$

$$- A_2 \Omega_0 [ \delta_1 + 2\delta_2 (\theta_0 + \theta_\Delta) ]$$

$$- [ \delta_1 + 2\delta_2 \theta_0 ] [ \Omega_0 (B_2 \dot{\beta} + C_2 w - C_2 \dot{\gamma}_C) + A_2 \Omega_0 ]$$

$$+ \delta_2 [ E_2 + 2A_2 \dot{\beta} + 2D_2 w - 2D_2 \dot{\gamma}_C ] + a \theta_\Delta \Omega_0 A_1 \} \quad (13)$$

where the following symbols have been used to represent the separate integrals involved in obtaining equation (13):

$$A_1 = \frac{\rho cb}{2} \int_0^{BR} r^2 v dr \quad (14)$$

$$B_1 = \frac{\rho cb}{2} \int_0^{BR} r^3 dr \quad (15)$$

$$C_1 = \frac{\rho cb}{2} \int_0^{BR} r^2 dr \quad (16)$$

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$$D_1 = \frac{\rho cb}{2} \int_0^{BR} vr dr \quad (17)$$

$$E_1 = \frac{\rho cb}{2} \int_0^{BR} v^2 r dr \quad (18)$$

$$F_1 = \frac{\rho cb}{2} \int_0^{BR} r dr \quad (19)$$

Use of the subscript 2 instead of 1 with the letter symbols in equation (14)-(19), as for example,  $A_2$ ,  $B_2$ , etc., means that the upper integration limit is  $R$  instead of  $BR$ .

As indicated in equation (13), the aerodynamic torque during the transient depends not only on  $\dot{Q}_A$ , the time variant disturbance in rotor speed, but also on  $\dot{\beta}$  and  $\dot{z}_c$ , the blade flapping and helicopter vertical motion, respectively. Thus, for evaluation of  $Q_A$ , as required for the solution of equation (1), it is necessary to write simultaneous equations defining the flapping and helicopter vertical motion. This is the subject of the next two sections of this report.

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## III THE BLADE CONING EQUATION

Consider the requirement from dynamical considerations for the equilibrium of moments about the flapping hinge.

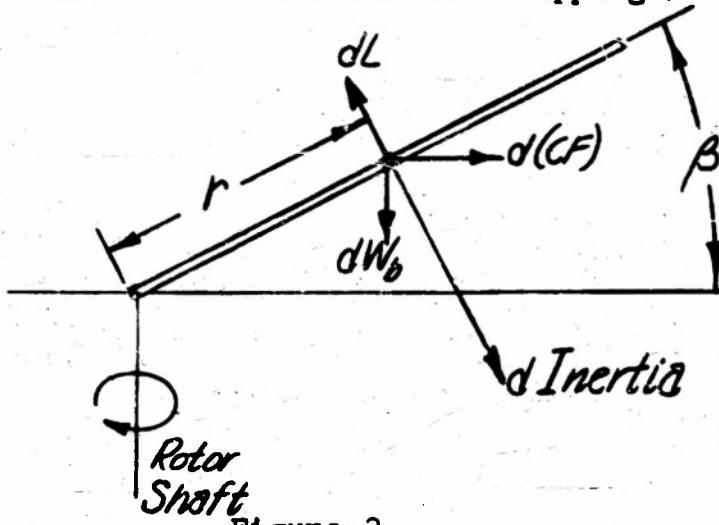


Figure 2

From the figure,

$$M_L - M_{Wb} - M_{CF} - M_I = 0 \quad (20)$$

where the individual terms are moments about the flapping hinge caused by the lift, blade weight, centrifugal, and inertia forces, respectively. Some of these can be evaluated by inspection:

$$M_{Wb} = \int_0^R m g r dr = mg R^2/2 \quad (21)$$

$$M_{CF} = \int_0^R m \Omega^2 r (r \beta) dr \quad (22)$$

$$\begin{aligned} &= \int_0^R m (\Omega_0 + \Omega_\Delta)^2 (\beta_0 + \beta_\Delta) r^2 dr \\ &= m (\Omega_0^2 + 2\Omega_0\Omega_\Delta) \beta_0 R^3/3 \end{aligned}$$

$$+ m \Omega_\Delta^2 \beta_\Delta R^3/3 \quad (23)$$

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$$M_I = \int_0^R (mr\ddot{\beta})rdr = m\ddot{\beta} R^3/3 \quad (24)$$

The lift moment is derived from

$$M_L = \int_0^R r \frac{dL}{dr} dr \quad (25)$$

$$dL = \frac{\rho a}{2} \left( \theta - \frac{U_p}{U_T} \right) U_T^2 dr \quad (26)$$

Substituting as before and integrating,

$$M_L = \frac{a}{b} \left[ \theta_0 B_1 (\Omega_0^2 + 2\Omega_0 \Omega_\Delta) + \theta_\Delta B_1 \Omega_0^2 - \Omega_0 (A_1 + B_1 \dot{\beta} + C_1 w - C_1 \dot{\gamma}_c) - A_1 \Omega_\Delta \right] \quad (27)$$

Consequently, equation (20) can be expanded to give, for the variation in flapping angle  $\beta$ ,

$$\begin{aligned} & \frac{a}{b} \left[ \theta_0 B_1 (\Omega_0^2 + 2\Omega_0 \Omega_\Delta) + \theta_\Delta B_1 \Omega_0^2 \right. \\ & \left. - \Omega_0 (A_1 + B_1 \dot{\beta} + C_1 w - C_1 \dot{\gamma}_c) - A_1 \Omega_\Delta \right] \\ & - mg R^2/2 - m(\Omega_0^2 + 2\Omega_0 \Omega_\Delta) \beta_0 R^3/3 \\ & - m\Omega_0^2 \beta_\Delta R^3/3 - m\ddot{\beta} R^3/3 = 0 \end{aligned} \quad (28)$$

where it might be noted that

$$\dot{\beta} = \dot{\beta}_1, \text{ and } \ddot{\beta} = \ddot{\beta}_1.$$

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#### IV VERTICAL MOTION OF THE HELICOPTER

The equation for vertical motion of the helicopter is simply:

$$W - T = \frac{W}{g} \ddot{z}_c \quad (29)$$

for which only the value of rotor 'thrust' is required to permit the solution to be obtained. In this case, the rotor 'thrust' is taken to be the net force transferred to the helicopter by the blade system. Thus, from figures (1) and (2), the elemental 'thrust' is given by

$$dT = dL - \phi dD - dI \quad (30)$$

where the blade weight has been omitted since it is considered as part of the helicopter weight  $W$  in equation (29). As is well known,  $\phi dD$  can be neglected in evaluating equation (30). Writing  $dI$  as

$$dI = mr\ddot{\beta}dr \quad (31)$$

and combining with equation (26), integrating and substituting in equation (29), there results

$$\begin{aligned} W - a & \left[ \theta_0 C_1 (\Omega_0^2 + 2\Omega_0 \Omega_\Delta) + \theta_\Delta \Omega_0^2 C_1 \right. \\ & \left. - \Omega_0 (D_1 + C_1 \dot{\beta} + F_w - F_z \dot{z}_c) - D_\Delta \Omega_\Delta \right] \\ & - m \ddot{\beta} \frac{R}{2} b - \frac{W}{g} \ddot{z}_c = 0 \end{aligned} \quad (32)$$

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V SUMMARY OF EQUATIONS

The torque, flapping, and vertical motion equations, equation (13), (28), and (32), respectively, constitute a set of three simultaneous differential equations and can be written in the following form:

$$Q_A = Q_0 + Q_w w + Q_{\dot{\beta}} \dot{\beta} + Q_{\theta_a} \theta_a + Q_{\dot{\theta}_a} \dot{\theta}_a + Q_{\dot{\beta}_c} \dot{\beta}_c \quad (33)$$

$$0 = M_0 + M_w w + M_{\dot{\beta}} \dot{\beta} + M_{\theta_a} \theta_a + M_{\dot{\theta}_a} \dot{\theta}_a + M_{\dot{\beta}_c} \dot{\beta}_c + M_{\ddot{\beta}} \ddot{\beta} \quad (34)$$

$$0 = T_0 + T_w w + T_{\dot{\beta}} \dot{\beta} + T_{\theta_a} \theta_a + T_{\dot{\theta}_a} \dot{\theta}_a + T_{\dot{\beta}_c} \dot{\beta}_c + T_{\ddot{\beta}_c} \ddot{\beta}_c + T_{\ddot{\beta}} \ddot{\beta} \quad (35)$$

The first term on the right hand side of these equations represents the steady-state term, and for purposes of solving equation (1), each of these can be ignored. However, these may be of interest in establishing the values of steady-state flapping angle, collective pitch angle, etc. The coefficients of the variables in equation (33)-(35) are obtained from the basic equations (13), (28), and (32), and together with the steady-state terms are listed below:

$$Q_0 = a(\theta_0 \Omega_0 A, -E_1) + B_2 \Omega_0^2 (\delta_0 + \delta_1 \theta_0 + \delta_2 \theta_0^2) - A_2 \Omega_0 (\delta_1 + 2\delta_2 \theta_0) + \delta_2 E_2$$

$$Q_w = a(\theta_0 \Omega_0 C_1 - 2D_1) - (\delta_1 + 2\delta_2 \theta_0) \Omega_0 C_2 + 2\delta_2 D_2$$

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$$Q_{\dot{\beta}} = \alpha(\theta_0 \Omega_0 B_1 - 2A_1) - (\delta_1 + 2\delta_2 \theta_0) \Omega_0 B_2 + 2\delta_2 A_2$$

$$Q_{\dot{\alpha}} = \alpha \Omega_0 A_1 + B_2 (\delta_1 + 2\delta_2 \theta_0) \Omega_0^2 - 2\delta_2 \Omega_0 A_2$$

$$Q_{\dot{\Omega}_0} = \alpha \theta_0 A_1 + 2B_2 (\delta_0 + \delta_1 \theta_0 + \delta_2 \theta_0^2) \Omega_0 \\ - (\delta_1 + 2\delta_2 \theta_0) A_2$$

$$Q_{\dot{\gamma}} = -\alpha(\theta_0 \Omega_0 C_1 - 2D_1) + (\delta_1 + 2\delta_2 \theta_0) \Omega_0 C_2 \\ - 2D_2 \delta_2$$

$$M_0 = \frac{\alpha}{b} (\theta_0 B_1 \Omega_0^2 - \Omega_0 A_1) - mg R_2^2 - m \Omega_0^2 \beta_0 R_3^3 = 0$$

$$M_{\dot{\alpha}} = -\alpha \Omega_0 C_1 / b$$

$$M_{\dot{\beta}} = -\alpha \Omega_0 B_1 / b$$

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$$M_{\theta_1} = a B_1 \Omega_0^2 / b$$

$$M_{\Omega_1} = 2 a B_1 \theta_0 \Omega_0 / b + a A_1 / b - 2 m \Omega_0 \beta_0 R^2 / 3$$

$$M_{\dot{\gamma}_0} = a \Omega_0 C_1 / b$$

$$M_{\beta} = - m \Omega_0^2 R^3 / 3$$

$$M_{\ddot{\beta}} = - m R^3 / 3$$

$$T_0 = W - a \theta_0 C_1 \Omega_0^2 - \Omega_0 D_1 = 0$$

$$T_w = \Omega_0 F_1 a$$

$$T_{\beta} = \Omega_0 C_1 a$$

$$T_{\theta_1} = - a \Omega_0^2 C_1$$

$$T_{\dot{\gamma}_0} = - 2 \theta_0 C_1 2 \Omega_0 + a D_1$$

$$T_{\ddot{\gamma}_0} = - a \Omega_0 F_1$$

$$T_{\ddot{\beta}} = - \frac{W}{8}$$

$$T_{\ddot{\beta}} = - m b R^2 / 2$$

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**VI SYMBOLS**

a slope of lift coefficient curve, 1/rad.  
b number of blades in rotor  
B tip loss factor  
c blade chord, ft.  
 $C_{D0}$  drag coefficient  
 $C_L$  lift coefficient  
 $dD$  drag force on the blade element, lbs.  
 $dL$  lift force on the blade element, lbs.  
 $d(CF)$  centrifugal force on blade element, lbs.  
 $dW_b$  weight of blade element, lbs.  
 $dI$  inertial force on blade element, lbs.  
 $dT$  blade element thrust (see equation 30)  
g acceleration of gravity, ft/sec<sup>2</sup>  
 $I_{eff}$  moment of inertia of rotating system, ft. lb. sec<sup>2</sup>  
 $M_{CF}$  centrifugal force moment about flapping hinge, ft. lb.  
 $M_I$  inertia moment about flapping hinge, ft. lb.  
 $M_L$  aerodynamic moment about flapping hinge, ft. lb.  
 $M_{W_b}$  blade weight moment about flapping hinge, ft. lb.  
m mass density of blade per foot of span, slugs/ft.  
 $Q_A$  change in rotor aerodynamic torque, ft. lb.  
 $Q_e$  change in available engine torque, ft. lbs.  
r radius of a blade element, ft.  
R rotor radius, ft.  
T integral of  $dT$  for b blades.